# 13. MATHEMATICS / MANIPULATIVES

## Children at Play: Mathematics / Manipulatives

Mathematics, like language, pervades all of human experience. In every culture, mathematical concepts are used in ordinary life: in measuring and keeping track of the passage of time; in practical activities like farming, construction, and cooking; and in every sort of commerce, from food shopping to global finance. Mathematics has provided the foundation for the astonishing technological progress that has taken our world from the industrial revolution into the space age and onward into the age of telecommunication. As we approach the twenty-first century, our children will live and work in a world driven by the exchange of information through technology. The key to economic self-sufficiency may well be mathematical competence. We must instill in our children a mathematical literacy that will continue after they leave school. To be successful in the larger world, they must be able to communicate mathematically and conceptualize problems in a mathematical framework.

Mathematics is an abstract system for organizing and ordering experience. Young children, however, think very concretely. Concepts such as quantity and ordination mean nothing to them if they do not have concrete objects to count and put in order. Therefore, young children must have opportunities to experience mathematical relationships through the manipulation of concrete objects, that is, they must play with things that they can count and sort. Such play achieves meaning for children with the supportive intervention of an adult facilitator or teacher. Early childhood teachers help children construct and become aware of mathematical concepts operating in their everyday lives. The learning and mastery of mathematical concepts does not come from workbook pages or paper and pencil tasks; instead, children construct enduring, useful mathematical knowledge and develop mathematical competence through directly interacting with the world around them. They internalize and reconstruct this knowledge through hands-on experiences with real-life activities.

In early conceptual development, the foundations of mathematics overlap with many other areas of knowledge, especially language and logic. For the sake of clarity and simplicity, the Step by Step Program defines early childhood math concepts as follows:

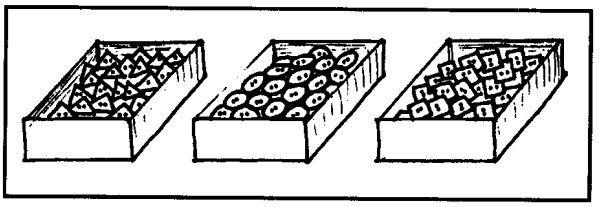
- **One-to-one correspondence** involves the distribution of related items in direct relation to each other, such as one cookie for each child, one pillow for each cot, or one chair for each child.

- **Seriation** involves the ability to put things in order, first by size (smallest to largest) and then by number. In order to do the latter, a child must recognize numerals and be able to assign a quantity to them.

- **Counting** involves the ability to demonstrate an understanding of number and amount. It involves the ability to answer the questions, "Which number is this?" and, "What comes next?"

- **Calculation** is the process of adding and subtracting, as it is experienced concretely.

- **Classification** involves the ability to sort objects by attribute (for example, color, shape, size). It involves the ability to answer the question, "Why do these belong together?"



- **Measurement** is the process of finding the number of standard units in an object.

- **Comparison** involves the ability to determine that one object is greater than, less than, or equal to another through measurement.

- **Geometry** is the study of spatial relationships. For young children this involves the exploration of objects and their relationships, as well as the recognition of shape and pattern.

- **Pattern** is a theme that connects mathematical topics. It encourages children to see relationships, find connections and make generalizations and predictions. Understanding the concept of patterns guides children into recognizing the implied predictability and repetition of patterns. For example:

  — Patterns represent a basic unit: yellow-red, yellow-red
  — Patterns expand: snap-clap, snap-snap-clap
  — Patterns of nature: fall, winter, spring, summer

Young children should experience regularity and repetition in motion, color, sound, position, quantity, and time. Patterns describe, extend, transfer, translate, and create patterns.

## *Integrating Mathematics into the Daily Schedule*

Daily classroom routines offer many occasions for concrete mathematical experiences. For example:

- At **arrival time**, hanging up coats can offer an experience in one-to-one correspondence: one hook or cubby for each coat.

- Taking **daily attendance** offers practice at counting and calculation: How many are here? How many are absent? How many are there in all? There are ten of us here, but there should be fourteen; how many are absent? Using photographs of the children present and absent makes the counting and calculating a concrete experience.

- **Breakfast time** provides children with another chance to experience one-to-one-correspondence: one placemat, one plate, one cup, and one napkin for each child.

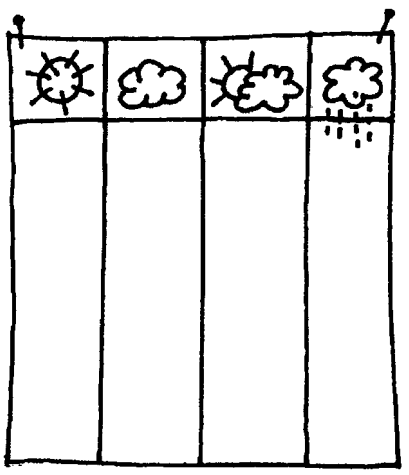
- **Circle time** offers the opportunity to read number stories and sing many number songs, recite nursery rhymes and sing chants for pattern recognition and a sense of rhythm. It also offers chances for counting children and classifying them by attribute: How many girls are there? How many people are wearing red? How many boys have white socks?

- **Outdoor time** features physical activity, which is an excellent way to give children concrete experiences in math. Any activity that involves rhythmic movement of the body can be used for counting: climbing or descending steps, when one number is said for each step; swinging, and counting each swing out loud; jumping or hopping; or bouncing a ball. In such activities, the child's whole body reinforces the meaning of the number. Rote counting and identification of numerals are meaningless without such concrete experiences.

- Even **toileting** can be a time for mathematical conversation. The teacher can introduce calculation, for example, by saying, "We have three sinks and four children. How many children are waiting?" Distributing towels for drying hands can be another opportunity for one-to-one correspondence.

The daily schedule offers a concrete experience in measuring the passage of time. Digital clocks that show the passage of minutes can be used to call children's attention to time periods. Standard clocks that show minutes and have a second hand are also useful. Other, more concrete time measuring devices can give more vivid experiences in measuring time. For example, a sand-filled three-minute egg timer can be used to alert children to prepare for transition from one activity to another. A kitchen timer that measures seconds, signaling the passage of time in both movement of the dial and the corresponding "tick" sound, adds similar drama to the passage of short intervals of time and can help impatient four-year-olds learn to increase their ability to wait.

Calendars and weather are popular topics at circle time in many early childhood classrooms. Standard calendars, however, are not very meaningful to young children, and rote repetition of the number of the day of the month and the year does not have much potential for helping children grasp the concept of time. A month is too long; the numeral for the day of the month and for the year far too abstract. On the other hand, most young children can grasp days of the week. Three- and four-year-olds can understand a calendar of weeks better than a calendar of months. Teachers can make a simple week-by-week calendar with each page showing seven days. Each day should be a different color, with all Mondays the same color, all Tuesdays a second color, etc. Label the days and differentiate the days that children are in school from the days they are not in school.



This calendar can be used to record the weather each day. The number of sunny days versus cloudy days can be calculated. The teacher can use pictures to indicate special events that are to take place during the week, and help the children count the days to that event. When a child is sick and must stay at home, her absence can be noted on the calendar, and the teacher can help the children count the number of days of her absence. This is particularly helpful when a child must be out of school for a hospital stay. Marking the day when he will return helps the children understand, within the concept of time appropriate to their developmental level, how long their friend will be out of school.

Field trips offer many opportunities for mathematical experiences. Marking trips on the calendar several days in advance gives children the experience of measuring time by counting days. The length of time to get from school to the destination can be noted and discussed. There should be one adult for every

two children on any field trip; children can calculate how many adults are needed. Any time children climb stairs, they can be counted.

Trips to stores are particularly rich sources of math experiences. For example, a shopping trip can precede a cooking activity. Planning can include calculation of how many of each item are needed, and a picture list can be developed. Actual purchases can involve counting and calculating with currency.

## *Integrating Mathematics with Other Activity Centers*

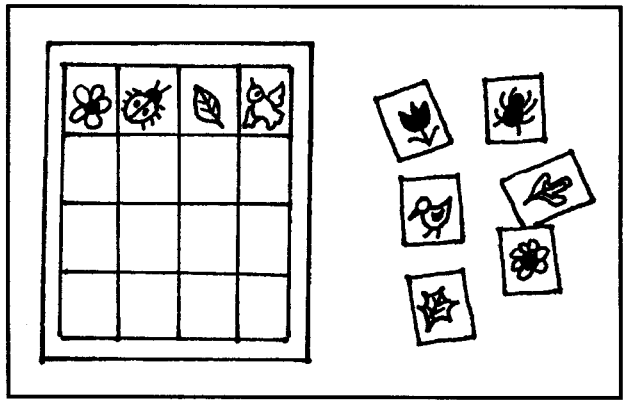
All the learning centers in an early childhood classroom offer opportunities for mathematical learning:

- **Dramatic Play Area** One-to-one correspondence can be reinforced in playing house, for example, dolls to beds or coffee cups to "mommies." Playing "store" offers opportunities to use play currency for simple calculation.

- **Literacy Corner** There should be a good stock of picture-number books with clear, simple numerals and interesting pictures to count.

- **Sand and Water Table (or Rice and Bean Table)** Filling and emptying a variety of containers gives children experience with the concepts of measurement and comparison.

- **Art Area** Art activities offer other opportunities for reinforcing one-to-one correspondence, for example, brushes to colors of paint or papers to children working in that area. Simple calculation can be introduced. For example, each child is using three crayons: How many crayons are we all using? Designing projects can involve geometry through the use of shapes and patterns.

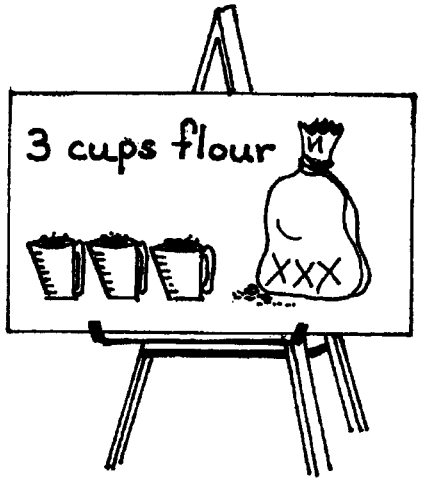


- **Manipulative/Table Toys** Manipulatives are small materials that most often are used at a table. These materials require children to use their eyes and hands to do or make something. Among the many examples of such materials are puzzles, pegs, Lotto, and Legos. Manipulatives are usually used by children individually rather than in pairs or in groups.

Everyday items such as dried beans, buttons, bottle caps, shells, and pebbles are ideal for counting and classification. Muffin tins or egg cartons make excellent containers for this activity. Peg boards and rubber bands offer both counting and geometric possibilities. Construction sets based on a unit and regular multiples of that unit give experience in all the math concepts (except time) emphasized in this activity center. Such sets are much more useful in helping children develop math concepts than materials that present numerals out of context, such as games and puzzles involving numeral identification.

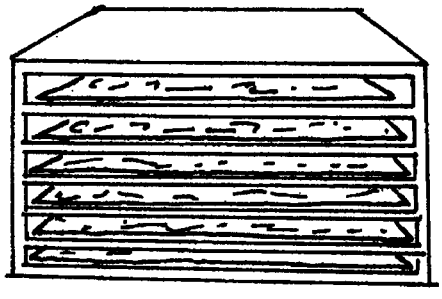
**Block Area** If only one commercial item can be purchased for a new early childhood classroom, it should be a set of wooden unit blocks. Working with such a set to make their own structures, children experience mathematical and geometric relationships at an intuitive level which provides the basis on which the abstract concepts of algebra and plane geometry will build.

**Cooking Area** Teachers can copy recipes onto chart-size paper, with measurements represented in pictures, for example, if the recipe calls for three cups of flour, there would be a picture of three cups and a bag of flour. *Although European recipes call for metric measurements, and dry ingredients are usually specified in grams, the early childhood teacher might consider converting to cups, since a measuring cup is far less abstract than grams measured on a scale.* Children can count ingredients as they are added to the mix, count how many times it is stirred, time the cooking process, use geometric shapes to form cookies and sandwiches, and calculate how many each child may eat.




## *Setting Up a Mathematics / Manipulatives Area*

A spacious area in the classroom, separate from the more active play areas like the Dramatic Play Center, should be set up for use of manipulative materials. Well-organized, open shelves should be used to display the materials. The materials should be clean and intact (puzzles should have all their pieces, pegboards should be shelved with containers of pegs, and Lotto cards should have all their tiles). Puzzle racks should be used to keep puzzles complete and separate.



The area should have tables and chairs and provide enough space for several children to use their materials comfortably and simultaneously without interfering with each other.

Materials from everyday life have as much potential for revealing mathematical concepts to young children as learning tools that are specifically designed for teaching mathematics. However, several items that offer systematic, concrete experiences in counting, ordination, calculation, and comparison can have a profound impact on children's grasp of mathematical concepts. These items include:

- **Inch Cubes**, such as Unifix Cubes, for counting and measuring activities.

- **Floor Numbers** Nonskid squares with the numbers 1 - 10 printed on them. These are valuable for gross motor games in which motor activity reinforces the experience of counting. These can be made with carpet or paper or taped on the floor. They also can be used for games involving ordination and numeral recognition.

- **Measuring Equipment** Concrete experiences with measurement do not require standard measuring devices; teacher-made or child discovered units can be used. Nonetheless, standard equipment can be helpful. Liquid and dry metric measuring cups in graduated sizes, measuring spoons, rulers, tape measures, scales, and thermometers should be available for use by the children or for demonstration. Simple balance scales are more meaningful than metered scales (such as bathroom or kitchen scales), because they give children the opportunity to see the comparison of two items and offer the possibility of comparing diverse items against a standard weight.

- **Number Lines** are helpful as children begin to understand the concepts of counting and ordination and to move toward performing simple calculations. A number line on the floor that is large enough to walk along offers the physical reinforcement of stepping out a calculation.

- **Parquetry Blocks** offer experiences in geometric patterning and sequencing. Blocks can be matched for color and shape, and designs can be created with them.

- **Attribute Blocks** These plastic blocks can be very useful in helping children discover mathematical concepts. The blocks are in three shapes, three thicknesses, three colors, and three sizes. One block might be a small, thick,

red triangle; another might be a large, thin, blue circle; a third might be a large, thin, yellow square. Children can sort them by one, two, or three attributes. Their simplest use is in concrete experience with geometric shapes. At the next level of sophistication, they offer children the experience of classifying by one attribute: size, shape, color, or thickness. At the next level, a child can classify by two attributes simultaneously. Moving from one attribute (all the thin ones or all the red ones) to two attributes (which ones are both thin and red?) introduces the child to symbolic logic. Symbolic thinking does not occur until children are between five and seven years old.

- **Coins** "Play money" and real coins in small denominations offer experiences in classification and calculation.

- **Learning Clock** The best clocks for learning to tell time have a large face with second, minute, and hour hands connected by visible gears. Numbers for the hours and marks for the minutes and seconds are clearly indicated. By manipulating the hands, children discover the relationship between seconds, minutes, and hours.

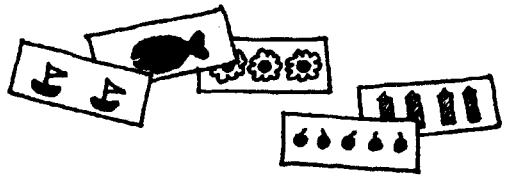
- **Puzzles** Puzzles help children focus on size and shape as well as on the relationship of parts to a whole. These are necessary concepts for both math and reading. Using puzzles, children become aware of clues (science). They classify ideas as they figure out where the green grass goes, where the blue sky is, and where the head, arms, and legs of the child in a puzzle picture fit. Classification is important in math, science, and reading.

  Working on puzzles also requires analyzing and testing ideas, another aspect of science. Some puzzles have pictures of vehicles, some of community workers, and some of animals. Analyzing the nature and environment of these scenes can contribute to concepts used in science and social studies. Puzzles with letters of the alphabet can contribute to reading readiness; puzzles with numerals can contribute to math concepts. This learning can be done at simple or more complex levels, as puzzles vary in complexity, depending upon the number and size of the pieces.

- **Pegboards and Pegs** Pegboards and pegs of different colors are also good for sorting and classifying. Math concepts are developed as children count pegs by cardinal numbers (one, two, three) or by ordinal numbers (first, second, third). By adding and subtracting pegs, they develop concepts such as "more than," "less than," and "the same as."

  Children may also develop patterns with pegs. Recognizing patterns is important in learning math and science, in reading readiness, and in developing creative artistic expression.

- **Lotto Games** These games come in varying types and degrees of difficulty. They may involve identification of colors, numerals, or pictures. All require observation, comparison, and matching skills, which are necessary in the development of science concepts and reading readiness. Lotto games that feature pictures of community workers contribute to the development of social studies concepts.



- **Lego or Duplo Blocks** These blocks offer endless possibilities for creating structures: buildings, towers, vehicles, and other objects. They also offer opportunities for children to experiment without needing to create anything objectively identifiable. Accessories for these small building blocks include wheels, people (family members and community workers), vehicles, and animals. Play with these materials contributes to the development of concepts related to math (patterning, cardinal and ordinal counting, understanding area, and dealing with comparative sizes).

  When turned into ramps, bridges, and tunnels, these blocks contribute to the development of concepts of balance, strength, and stability. They provide an introduction to understanding patterns, symmetry, design and architecture.

- **Dominos** Dominos come in many forms. With some, children have to match geometric shapes. With others, children work with pictures or colors. Still others require that children match numbers of dots or numerals. (Some dominos have a numeral on one side and the corresponding number of dots on the other.) As with Lotto games, dominos require children to observe, compare, identify, and match. All of these skills are important in the study of science and in reading readiness.

- **Colored Beads** When children put beads of different colors on a string, they learn cardinal and ordinal numbers as well as patterning and color discrimination.

- **Colored Magnetic Pieces** A metal tray with colored magnetic pieces in different geometric shapes and varying sizes permits children to create patterns of their choosing. The activity provides opportunities for developing number theories and geometric concepts, increasing a child's understanding of colors, creative expression, and dealing with the scientific concept of magnetism.

## The Teaching Team's Role

The first role of the teaching team is to ensure that the classroom environment is full of materials that offer a variety of opportunities for developing mathematical thinking skills. In conjunction with this, the teacher must practice his skills at catching the "teachable moment." Rather than imposing math lessons on children, teachers should observe child-initiated play and take advantage of opportunities to introduce or discuss a math concept. Thus, the teacher acts as facilitator to the children.

This approach is particularly successful when the teacher helps children arrive at a mathematical solution to a problem that a child has presented. A child may have a problem, for example, with a building she is trying to construct with blocks. The teacher can act as an adviser, suggesting various sizes and arrangements of blocks to achieve what the child wants. Even a social problem can have a mathematical solution. If, for example, two children have a dispute over the use of a building set, the teacher can ask them to divide the pieces of the set between them so that each child has the same number of pieces. Too few tricycles for all the children who want to ride may lead to the use of a timer to solve a social problem.

The teacher helps the child expand concrete experience by modeling mathematical language. For example, in an activity involving comparison, he can introduce the phrases, "greater than," "less than," and "equal to." All of the opportunities for meaningful counting described above illustrate this math-language role modeling. To reinforce a particular mathematical experience, the teacher can frame mathematical questions. In setting the table, for example, she can ask, "How many napkins do we need?" or "How many more spoons do we need?"

The more the teacher observes what the children have chosen to do, the more she can use their chosen activities to introduce or reinforce mathematical concepts.

## Activities and Projects

Children can gain an understanding of the basic concepts of one-to-one correspondence, counting, ordination, calculation, classification, measurement, comparison, geometry, and time, as they manipulate objects in their environment.

- **Counting** Counting is meaningful only when it applies to concrete objects and is most meaningful when it solves a problem that interests a child. However, simple teacher-made card games that involve counting cards out and matching designs and teacher-made board games that involve counting "moves" on a board, counting dots on dice, or using a number spinner, are enjoyable activities that reinforce the concepts of number and amount.

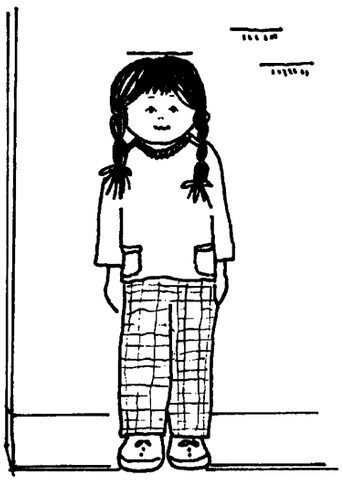
- **Ordination** Stairs present an excellent opportunity to help children learn to put things in order by number. Nonskid number squares can be placed in order going up the stairs ("1" on the first step, "2" on the second, and so forth). The teacher can then scramble the number squares and ask the children to reorder them, on the stairs. Each child can choose a number and place it on the appropriate stair. To avoid falls, this activity should involve only two or three children at a time and must be carefully supervised. Another activity for introducing ordination uses a teacher-made set of cardboard tubes in graduated lengths. Children can work in pairs to stand them in order from the shortest to the longest. Number the tubes to reinforce the concept of ordination.

- **Calculation** Simple addition and subtraction can be introduced in dozens of routine activities. Computation can often solve a concrete problem presented by the children ("How many more cookies do we need so that everyone has one?"). Any sorting activity can be made into a computational activity involving addition or subtraction: for example, "Take all the small buttons out of the cup of red ones. How many are left?" or "Put the white buttons with the black buttons. How many do you have?"

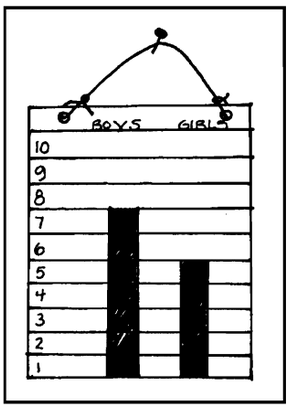
- **Classification** Sorting is most meaningful as a problem-solving technique in a child-initiated activity. However, teacher-made classification activities can reinforce the concept. For example, a box of buttons can be sorted into groups by shape, color, size. Children can then be encouraged to describe the criteria for classification, naming how the buttons are alike and different. ("These buttons are red; those are blue.") Helping the child to reclassify the buttons by a different attribute introduces the child to logical relationships. ("These red buttons are square; those red buttons are round.")

- **Measurement** Cooking activities provide a natural context for the introduction of measurement. Other measurement activities include comparing children's heights with marks on the wall and comparing weights using a balance scale. Floor tiles can be used as a measuring unit

for all kinds of things, from children's height to the length and width of furniture to the length of ribbon streamers for a dance activity.

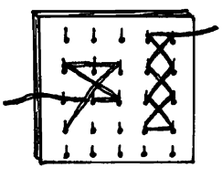


- **Comparison** Graphing is a way to combine counting and measurement to make concrete mathematical comparisons. A week calendar could be transformed into a weather graph, with sunny days compared with cloudy days. Another graph could compare the number of boys to the number of girls in the class. On any given day, numbers of children wearing certain colors could be graphed. Other topics include favorite foods, pets, lost teeth, birthdays, and shoe sizes.




- **Time** Children can construct simple time-measuring devices such as a drip clock. A clear plastic bottle (such as a soda bottle) works well for this project. The teacher makes a small hole in the bottom, and suspends the bottle over a container to catch the drips. At regular intervals (preferably short, like one minute), she and the children note and mark how much water has dripped out. The bottle then can be used to time such regular classroom activities as transition time or to time how long each child may play with a coveted toy.

- **Geometry** An excellent activity for discovering the properties of geometric shapes is a teacher-made nail board. Small nails are pounded part-way into a square board to form a grid. Using rubber bands stretched from nail to nail, children can experiment with the construction of various shapes. Different colors of rubber bands allow overlap and comparison of shapes.



## *Summary*

Early experiences with math concepts build the foundation for higher-level mathematical thinking. Using mathematical thinking to solve concrete, everyday problems will give children confidence in their mathematical skills. A strong foundation in math ensures that, as adults, they will be able to apply mathematical knowledge in practical situations as well as use that knowledge to participate successfully in the technologically complex global community.