CHAPTER 12

Binomial Distributions*

A basketball player shoots 5 free throws. How many does she make? A new treatment for pancreatic cancer is tried on 250 patients. How many survive for five years? You plant 10 dogwood trees. How many live through the winter? In all these situations, we want a probability model for a count of successful outcomes.

The binomial setting and binomial distributions

The distribution of a count depends on how the data are produced. Here is a common situation.

THE BINOMIAL SETTING

1. There are a fixed number $n$ of observations.
2. The $n$ observations are all independent. That is, knowing the result of one observation tells you nothing about the other observations.
3. Each observation falls into one of just two categories, which for convenience we call “success” and “failure.”
4. The probability of a success, call it $p$, is the same for each observation.

*This more advanced chapter concerns a special topic in probability. It is not needed to read the rest of the book.
Think of tossing a coin $n$ times as an example of the binomial setting. Each toss gives either heads or tails. Knowing the outcome of one toss doesn’t tell us anything about other tosses, so the $n$ tosses are independent. If we call heads a success, then $p$ is the probability of a head and remains the same as long as we toss the same coin. The number of heads we count is a random variable $X$. The distribution of $X$ is called a binomial distribution.

**BINOMIAL DISTRIBUTION**

The distribution of the count $X$ of successes in the binomial setting is the binomial distribution with parameters $n$ and $p$. The parameter $n$ is the number of observations, and $p$ is the probability of a success on any one observation. The possible values of $X$ are the whole numbers from 0 to $n$.

The binomial distributions are an important class of probability distributions. Pay attention to the binomial setting, because not all counts have binomial distributions.

**EXAMPLE 12.1 Blood types**

Genetics says that children receive genes from their parents independently. Each child of a particular pair of parents has probability $0.25$ of having type O blood. If these parents have 5 children, the number who have type O blood is the count $X$ of successes in 5 independent trials with probability $0.25$ of a success on each trial. So $X$ has the binomial distribution with $n = 5$ and $p = 0.25$.

**EXAMPLE 12.2 Dealing cards**

Deal 10 cards from a shuffled deck and count the number $X$ of red cards. There are 10 observations, and each gives either a red or a black card. A “success” is a red card. But the observations are not independent. If the first card is black, the second is more likely to be red because there are more red cards than black cards left in the deck. The count $X$ does not have a binomial distribution.

**Binomial distributions in statistical sampling**

The binomial distributions are important in statistics when we wish to make inferences about the proportion $p$ of “successes” in a population. Here is a typical example.

**EXAMPLE 12.3 Choosing an SRS**

An engineer chooses an SRS of 10 switches from a shipment of 10,000 switches. Suppose that (unknown to the engineer) 10% of the switches in the shipment are bad. The engineer counts the number $X$ of bad switches in the sample.
CHAPTER 12 • Binomial Distributions

This is not quite a binomial setting. Just as removing one card in Example 12.2 changes the makeup of the deck, removing one switch changes the proportion of bad switches remaining in the shipment. So the state of the second switch chosen is not independent of the first. But removing one switch from a shipment of 10,000 changes the makeup of the remaining 9999 switches very little. In practice, the distribution of $X$ is very close to the binomial distribution with $n = 10$ and $p = 0.1$.

Example 12.3 shows how we can use the binomial distributions in the statistical setting of selecting an SRS. When the population is much larger than the sample, a count of successes in an SRS of size $n$ has approximately the binomial distribution with $n$ equal to the sample size and $p$ equal to the proportion of successes in the population.

**SAMPLING DISTRIBUTION OF A COUNT**

Choose an SRS of size $n$ from a population with proportion $p$ of successes. When the population is much larger than the sample, the count $X$ of successes in the sample has approximately the binomial distribution with parameters $n$ and $p$.

**APPLY YOUR KNOWLEDGE**

In each of Exercises 12.1 to 12.3, $X$ is a count. Does $X$ have a binomial distribution? Give your reasons in each case.

12.1 M&Ms. Forty percent of all milk chocolate M&M candies are either red or yellow. $X$ is the count of red or yellow candies in a package of 40.

12.2 More M&Ms. You choose M&M candies from a package until you get the first red or yellow. $X$ is the number you choose before you stop.

12.3 Computer instruction. A student studies binomial distributions using computer-assisted instruction. After the lesson, the computer presents 10 problems. The student solves each problem and enters her answer. The computer gives additional instruction between problems if the answer is wrong. The count $X$ is the number of problems that the student gets right.

12.4 I can’t relax. Opinion polls find that 14% of Americans “never have time to relax.” If you take an SRS of 500 adults, what is the approximate distribution of the number in your sample who say they never have time to relax?

**Binomial probabilities**

We can find a formula for the probability that a binomial random variable takes any value by adding probabilities for the different ways of getting exactly that many successes in $n$ observations. Here is the example we will use to show the idea.
EXAMPLE 12.4 Inheriting blood type

Each child born to a particular set of parents has probability 0.25 of having blood type O. If these parents have 5 children, what is the probability that exactly 2 of them have type O blood?

The count of children with type O blood is a binomial random variable $X$ with $n = 5$ tries and probability $p = 0.25$ of a success on each try. We want $P(X = 2)$.

Because the method doesn’t depend on the specific example, let’s use “$S$” for success and “$F$” for failure for short. Do the work in two steps.

Step 1. Find the probability that a specific 2 of the 5 tries, say the first and the third, give successes. This is the outcome $SFSFF$. Because tries are independent, the multiplication rule for independent events applies. The probability we want is

$$P(SFSFF) = P(S)P(F)P(S)P(F)P(F)$$

$$= (0.25)(0.75)(0.25)(0.75)(0.75)$$

$$= (0.25)^2(0.75)^3$$

Step 2. Observe that the probability of any one arrangement of 2 S’s and 3 F’s has this same probability. This is true because we multiply together 0.25 twice and 0.75 three times whenever we have 2 S’s and 3 F’s. The probability that $X = 2$ is the probability of getting 2 S’s and 3 F’s in any arrangement whatsoever. Here are all the possible arrangements:

SSFFF SFSFF SFFSF SFFFS FSFSS SFSSF FFSSF FFSFS FFFSS

There are 10 of them, all with the same probability. The overall probability of 2 successes is therefore

$$P(X = 2) = 10(0.25)^2(0.75)^3 = 0.2637$$

The pattern of this calculation works for any binomial probability. To use it, we must count the number of arrangements of $k$ successes in $n$ observations.

We use the following fact to do the counting without actually listing all the arrangements.

**BINOMIAL COEFFICIENT**

The number of ways of arranging $k$ successes among $n$ observations is given by the binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

for $k = 0, 1, 2, \ldots, n$. 
CHAPTER 12 • Binomial Distributions

factorial The formula for binomial coefficients uses the factorial notation. For any positive whole number \( n \), its factorial \( n! \) is

\[
n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1
\]

Also, \( 0! = 1 \).

The larger of the two factorials in the denominator of a binomial coefficient will cancel much of the \( n! \) in the numerator. For example, the binomial coefficient we need for Example 12.4 is

\[
\binom{5}{2} = \frac{5!}{2! \cdot 3!} = \frac{(5)(4)(3)(2)(1)}{(2)(1)} \times \frac{(3)(2)(1)}{3} = \frac{(5)(4)}{2} = 10
\]

The notation \( \binom{n}{k} \) is not related to the fraction \( \frac{n}{k} \). A helpful way to remember its meaning is to read it as “binomial coefficient \( n \) choose \( k \).” Binomial coefficients have many uses in mathematics, but we are interested in them only as an aid to finding binomial probabilities. The binomial coefficient \( \binom{n}{k} \) counts the number of different ways in which \( k \) successes can be arranged among \( n \) observations.

The binomial probability \( P(X = k) \) is this count multiplied by the probability of any specific arrangement of the \( k \) successes. Here is the result we seek.

**BINOMIAL PROBABILITY**

If \( X \) has the binomial distribution with \( n \) observations and probability \( p \) of success on each observation, the possible values of \( X \) are 0, 1, 2, \ldots, \( n \). If \( k \) is any one of these values,

\[
P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}
\]

**EXAMPLE 12.5** Inspecting switches

The number \( X \) of switches that fail inspection in Example 12.3 has approximately the binomial distribution with \( n = 10 \) and \( p = 0.1 \).

The probability that no more than 1 switch fails is

\[
P(X \leq 1) = P(X = 1) + P(X = 0) = \binom{10}{1} (0.1)^1 (0.9)^9 + \binom{10}{0} (0.1)^0 (0.9)^{10} = \frac{10}{10} (0.1)(0.3874) + \frac{10}{10} (1)(0.3487) = (10)(0.1)(0.3874) + (1)(1)(0.3487) = 0.3874 + 0.3487 = 0.7361
\]
This calculation uses the facts that $0! = 1$ and that $a^0 = 1$ for any number $a$ other than 0. We see that about 74% of all samples will contain no more than 1 bad switch. In fact, 35% of the samples will contain no bad switches. A sample of size 10 cannot be trusted to alert the engineer to the presence of unacceptable items in the shipment.

**Using technology**

The binomial probability formula is awkward to use, particularly for the probabilities of events that contain many outcomes. You can find tables of binomial probabilities $P(X = k)$ and cumulative probabilities $P(X \leq k)$ for selected values of $n$ and $p$.

The most efficient way to do binomial calculations is to use technology. Figure 12.1 shows output for the calculation in Example 12.5 from a statistical software program, a spreadsheet, and a graphing calculator. We asked all three to give cumulative probabilities. Minitab and the TI-83 Plus have menu
entries for binomial cumulative probabilities. Excel has no menu entry, but the worksheet function BINOMDIST is available. All three outputs agree with the result 0.7361 of Example 12.5.

**APPLY YOUR KNOWLEDGE**

12.5 Inheriting blood type. If the parents in Example 12.4 have 5 children, the number who have type O blood is a random variable \( X \) that has the binomial distribution with \( n = 5 \) and \( p = 0.25 \).

(a) What are the possible values of \( X \)?

(b) Find the probability of each value of \( X \). Draw a histogram to display this distribution. (Because probabilities are long-run proportions, a histogram with the probabilities as the heights of the bars shows what the distribution of \( X \) would be in very many repetitions.)

12.6 Random-digit dialing. When an opinion poll or telemarketer calls residential telephone numbers at random, 20% of the calls reach a live person. You watch the random dialing machine make 15 calls. The number that reach a person has the binomial distribution with \( n = 15 \) and \( p = 0.2 \).

(a) What is the probability that exactly 3 calls reach a person?

(b) What is the probability that 3 or fewer calls reach a person?

12.7 Tax returns. The Internal Revenue Service reports that 8% of individual tax returns in 2000 showed an adjusted gross income of $100,000 or more. A random audit chooses 20 tax returns for careful study. What is the probability that more than 1 return shows an income of $100,000 or more? (Hint: It is easier to first find the probability that only 0 or 1 of the returns chosen shows an income this high.)

**Binomial mean and standard deviation**

If a count \( X \) has the binomial distribution based on \( n \) observations with probability \( p \) of success, what is its mean \( \mu \)? That is, in very many repetitions of the binomial setting, what will be the average count of successes? We can guess the answer. If a basketball player makes 80% of her free throws, the mean number made in 10 tries should be 80% of 10, or 8. In general, the mean of a binomial distribution should be \( \mu = np \). Here are the facts.

**BINOMIAL MEAN AND STANDARD DEVIATION**

If a count \( X \) has the binomial distribution with number of observations \( n \) and probability of success \( p \), the mean and standard deviation of \( X \) are

\[ \mu = np \]
\[ \sigma = \sqrt{np(1-p)} \]
 Binomial mean and standard deviation

Figure 12.2  Probability histogram for the binomial distribution with \( n = 10 \) and \( p = 0.1 \).

Remember that these short formulas are good only for binomial distributions. They can’t be used for other distributions.

EXAMPLE 12.6  Inspecting switches

Continuing Example 12.5, the count \( X \) of bad switches is binomial with \( n = 10 \) and \( p = 0.1 \). The histogram in Figure 12.2 displays this probability distribution. (Because probabilities are long-run proportions, using probabilities as the heights of the bars shows what the distribution of \( X \) would be in very many repetitions.) The distribution is strongly skewed. Although \( X \) can take any whole-number value from 0 to 10, the probabilities of values larger than 5 are so small that they do not appear in the histogram.

The mean and standard deviation of the binomial distribution in Figure 12.2 are

\[
\mu = np = (10)(0.1) = 1
\]

\[
\sigma = \sqrt{np(1 - p)} = \sqrt{(10)(0.1)(0.9)} = \sqrt{0.9} = 0.9487
\]

The mean is marked on the probability histogram in Figure 12.2.

APPLY YOUR KNOWLEDGE

12.8  Inheriting blood type. What are the mean and standard deviation of the number of children with type O blood in Exercise 12.5? Mark the location of the mean on the probability histogram you made in that exercise.
12.9 Random-digit dialing

(a) What is the mean number of calls that reach a person in Exercise 12.6?

(b) What is the standard deviation $\sigma$ of the count of calls that reach a person?

(c) If calls are made to New York City rather than nationally, the probability that a call reaches a person is only $p = 0.08$. What is $\sigma$ for this $p$? What is $\sigma$ if $p = 0.01$? What does your work show about the behavior of the standard deviation of a binomial distribution as the probability of a success gets closer to 0?

12.10 Tax returns

(a) What is the mean number of returns showing at least $100,000 of income among the 20 returns chosen in Exercise 12.7?

(b) What is the standard deviation $\sigma$ of the number of returns with income at least $100,000?

(c) The probability that a return shows income less than $50,000 is 0.72. What is $\sigma$ for the number of such returns in a sample of 20? The probability of income less than $200,000 is 0.98. What is $\sigma$ for the count of these returns? What does your work show about the behavior of the standard deviation of a binomial distribution as the probability $p$ of success gets closer to 1?

The Normal approximation to binomial distributions

The formula for binomial probabilities becomes awkward as the number of trials $n$ increases. You can use software or a statistical calculator to handle some problems for which the formula is not practical. Here is another alternative: as the number of trials $n$ gets larger, the binomial distribution gets close to a Normal distribution. When $n$ is large, we can use Normal probability calculations to approximate hard-to-calculate binomial probabilities.

EXAMPLE 12.7 Attitudes toward shopping

Are attitudes toward shopping changing? Sample surveys show that fewer people enjoy shopping than in the past. A survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." The population that the poll wants to draw conclusions about is all U.S. residents aged 18 and over. Suppose that in fact 60% of all adult U.S. residents would say "Agree" if asked the same question. What is the probability that 1520 or more of the sample agree?

Because there are more than 210 million adults, we can take the responses of 2500 randomly chosen adults to be independent. So the number in our sample who agree that shopping is frustrating is a random variable $X$ having the binomial distribution with $n = 2500$ and $p = 0.6$. To find the probability that
at least 1520 of the people in the sample find shopping frustrating, we must add the binomial probabilities of all outcomes from \( X = 1520 \) to \( X = 2500 \). This isn’t practical. Here are three ways to do this problem.

1. Use technology, as in Figure 12.1. The result is
   \[
P(X \geq 1520) = 0.2131\]

2. We can simulate a large number of repetitions of the sample. Figure 12.3 displays a histogram of the counts \( X \) from 1000 samples of size 2500 when the truth about the population is \( p = 0.6 \). Because 221 of these 1000 samples have \( X \) at least 1520, the probability estimated from the simulation is
   \[
P(X \geq 1520) = \frac{221}{1000} = 0.221\]

3. Both of the previous methods require software. Instead, look at the Normal curve in Figure 12.3. This is the density curve of the Normal distribution with the same mean and standard deviation as the binomial variable \( X \):
   \[
   \mu = np = (2500)(0.6) = 1500
   \]
   \[
   \sigma = \sqrt{np(1-p)} = \sqrt{(2500)(0.6)(0.4)} = 24.49
   \]
   As the figure shows, this Normal distribution approximates the binomial distribution quite well. So we can do a Normal calculation.
CHAPTER 12 • Binomial Distributions

EXAMPLE 12.8 Normal calculation of a binomial probability

If we act as though the count \( X \) has the \( N(1500, 24.49) \) distribution, here is the probability we want, using Table A:

\[
P(X \geq 1520) = P\left(\frac{X - 1500}{24.49} \geq \frac{1520 - 1500}{24.49}\right)
\]

\[
= P(Z \geq 0.82)
\]

\[
= 1 - 0.7939 = 0.2061
\]

The Normal approximation 0.2061 differs from the software result 0.2131 by only 0.007.

NORMAL APPROXIMATION FOR BINOMIAL DISTRIBUTIONS

Suppose that a count \( X \) has the binomial distribution with \( n \) trials and success probability \( p \). When \( n \) is large, the distribution of \( X \) is approximately Normal, \( N(np, \sqrt{np(1-p)}) \).

As a rule of thumb, we will use the Normal approximation when \( np \geq 10 \) and \( n(1-p) \geq 10 \).

The Normal approximation is easy to remember because it says that \( X \) is Normal with its binomial mean and standard deviation. The accuracy of the Normal approximation improves as the sample size \( n \) increases. It is most accurate for any fixed \( n \) when \( p \) is close to 1/2 and least accurate when \( p \) is near 0 or 1. Whether or not you use the Normal approximation should depend on how accurate your calculations need to be. For most statistical purposes great accuracy is not required. Our "rule of thumb" for use of the Normal approximation reflects this judgment.

APPLY YOUR KNOWLEDGE

12.11 Using Benford’s law. According to Benford’s law (Example 9.5, page 231) the probability that the first digit of the amount of a randomly chosen invoice is a 1 or a 2 is 0.477. You examine 90 invoices from a vendor and find that 29 have first digits 1 or 2. If Benford’s law holds, the count of 1s and 2s will have the binomial distribution with \( n = 90 \) and \( p = 0.477 \). Too few 1s and 2s suggests fraud. What is the approximate probability of 29 or fewer if the invoices follow Benford’s law? Do you suspect that the invoice amounts are not genuine?

12.12 Mark McGwire’s home runs. In 1998, Mark McGwire of the St. Louis Cardinals hit 70 home runs, a new major league record. Was this feat as surprising as most of us thought? In the three seasons before 1998, McGwire hit a home run in 11.6% of his times at bat. He went to bat 509 times in 1998. McGwire’s home run count in 509 times at
bat has approximately the binomial distribution with \( n = 509 \) and \( p = 0.116 \). What is the mean number of home runs he will hit in 509 times at bat? What is the probability of 70 or more home runs? (Use the Normal approximation.)

12.13 Checking for survey errors. One way of checking the effect of undercoverage, nonresponse, and other sources of error in a sample survey is to compare the sample with known facts about the population. About 12% of American adults are black. The number \( X \) of blacks in a random sample of 1500 adults should therefore vary with the binomial \((n = 1500, p = 0.12)\) distribution.

(a) What are the mean and standard deviation of \( X \)?
(b) Use the Normal approximation to find the probability that the sample will contain 170 or fewer blacks. Be sure to check that you can safely use the approximation.

**Chapter 12 Summary**

A count \( X \) of successes has a binomial distribution in the binomial setting: there are \( n \) observations; the observations are independent of each other; each observation results in a success or a failure; and each observation has the same probability \( p \) of a success.

The binomial distribution with \( n \) observations and probability \( p \) of success gives a good approximation to the sampling distribution of the count of successes in an SRS of size \( n \) from a large population containing proportion \( p \) of successes.

If \( X \) has the binomial distribution with parameters \( n \) and \( p \), the possible values of \( X \) are the whole numbers \( 0, 1, 2, \ldots, n \). The binomial probability that \( X \) takes any value is

\[
P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}
\]

Binomial probabilities in practice are best found using software.

The binomial coefficient

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

counts the number of ways \( k \) successes can be arranged among \( n \) observations. Here the factorial \( n! \) is

\[
n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1
\]

for positive whole numbers \( n \), and \( 0! = 1 \).

The mean and standard deviation of a binomial count \( X \) are

\[
\mu = np \\
\sigma = \sqrt{np(1-p)}
\]
CHAPTER 12 • Binomial Distributions

The Normal approximation to the binomial distribution says that if \( X \) is a count having the binomial distribution with parameters \( n \) and \( p \), then when \( n \) is large, \( X \) is approximately \( N(np, \sqrt{np(1-p)}) \). We will use this approximation when \( np \geq 10 \) and \( n(1-p) \geq 10 \).

**Chapter 12 EXERCISES**

12.14 Binomial setting! In each situation below, is it reasonable to use a binomial distribution for the random variable \( X \)? Give reasons for your answer in each case.

(a) An auto manufacturer chooses one car from each hour’s production for a detailed quality inspection. One variable recorded is the count \( X \) of finish defects (dimples, ripples, etc.) in the car’s paint.

(b) The pool of potential jurors for a murder case contains 100 persons chosen at random from the adult residents of a large city. Each person in the pool is asked whether he or she opposes the death penalty; \( X \) is the number who say “Yes.”

(c) Joe buys a ticket in his state’s “Pick 3” lottery game every week; \( X \) is the number of times in a year that he wins a prize.

12.15 Binomial setting! In each of the following cases, decide whether or not a binomial distribution is an appropriate model, and give your reasons.

(a) Fifty students are taught about binomial distributions by a television program. After completing their study, all students take the same examination. The number of students who pass is counted.

(b) A student studies binomial distributions using computer-assisted instruction. After the initial instruction is completed, the computer presents 10 problems. The student solves each problem and enters the answer; the computer gives additional instruction between problems if the student’s answer is wrong. The number of problems that the student solves correctly is counted.

(c) A chemist repeats a solubility test 10 times on the same substance. Each test is conducted at a temperature \( 10^\circ \) higher than the previous test. She counts the number of times that the substance dissolves completely.

12.16 Random digits. Each entry in a table of random digits like Table B has probability 0.1 of being a 0, and digits are independent of each other.

(a) What is the probability that a group of five digits from the table will contain at least one 0?

(b) What is the mean number of 0s in lines 40 digits long?
12.17 Universal donors. People with type O-negative blood are universal donors whose blood can be safely given to anyone. Only 7.2% of the population have O-negative blood. A blood center is visited by 20 donors in an afternoon. What is the probability that there are at least 2 universal donors among them?

12.18 Testing ESP. In a test for ESP (extrasensory perception), a subject is told that cards the experimenter can see but he cannot contain either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the subject names the shape on the card. A subject who is just guessing has probability 0.25 of guessing correctly on each card.

(a) The count of correct guesses in 20 cards has a binomial distribution. What are n and p?
(b) What is the mean number of correct guesses in many repetitions?
(c) What is the probability of exactly 5 correct guesses?

12.19 Random stock prices. A believer in the “random walk” theory of stock markets thinks that an index of stock prices has probability 0.65 of increasing in any year. Moreover, the change in the index in any given year is not influenced by whether it rose or fell in earlier years. Let X be the number of years among the next 5 years in which the index rises.

(a) X has a binomial distribution. What are n and p?
(b) What are the possible values that X can take?
(c) Find the probability of each value of X. Draw a probability histogram for the distribution of X.
(d) What are the mean and standard deviation of this distribution? Mark the location of the mean on your histogram.

12.20 How many cars? Twenty percent of American households own three or more motor vehicles. You choose 12 households at random.

(a) What is the probability that none of the chosen households owns three or more vehicles?
(b) What is the probability that at least one household owns three or more vehicles?
(c) What is the probability that your sample count is greater than the mean?

12.21 False positives in testing for HIV. The common test for the presence in the blood of antibodies to HIV, the virus that causes AIDS, gives a positive result with probability about 0.006 when a person who is free of HIV antibodies is tested. A clinic tests 1000 people who are all free of HIV antibodies.

(a) What is the mean number of positive tests?
(b) What is the distribution of the number of positive tests?
(c) You cannot safely use the Normal approximation for this distribution. Explain why.

12.22 Multiple-choice tests. Here is a simple probability model for multiple-choice tests. Suppose that each student has probability $p$ of correctly answering a question chosen at random from a universe of possible questions. (A strong student has a higher $p$ than a weak student.) Answers to different questions are independent. Jodi is a good student for whom $p = 0.75$.

(a) Use the Normal approximation to find the probability that Jodi scores 70% or lower on a 100-question test.
(b) If the test contains 250 questions, what is the probability that Jodi will score 70% or lower?

12.23 Planning a survey. You are planning a sample survey of small businesses in your area. You will choose an SRS of businesses listed in the telephone book's Yellow Pages. Experience shows that only about half the businesses you contact will respond.

(a) If you contact 150 businesses, it is reasonable to use the binomial distribution with $n = 150$ and $p = 0.5$ for the number $X$ who respond. Explain why.
(b) What is the mean number who respond to surveys like yours?
(c) What is the probability that 70 or fewer will respond? (Use the Normal approximation.)
(d) How large a sample must you take to increase the mean number of respondents to 100?

12.24 Survey demographics. According to the Census Bureau, 9.96% of American adults (age 18 and over) are Hispanics. An opinion poll plans to contact an SRS of 1200 adults.

(a) What is the mean number of Hispanics in such samples?
(b) What is the probability that the sample will contain fewer than 100 Hispanics? (Use the Normal approximation.)

12.25 Leaking gas tanks. Leakage from underground gasoline tanks at service stations can damage the environment. It is estimated that 25% of these tanks leak. You examine 15 tanks chosen at random, independently of each other.

(a) What is the mean number of leaking tanks in such samples of 15?
(b) What is the probability that 10 or more of the 15 tanks leak?
(c) Now you do a larger study, examining a random sample of 1000 tanks nationally. What is the probability that at least 275 of these tanks are leaking?

12.26 Language study. Of American high school students, 41% are studying a language other than English. An opinion poll plans to ask
high school students about foreign affairs. Perhaps language study will influence attitudes. If the poll interviews an SRS of 500 students, what is the probability that between 35% and 50% of the sample are studying a foreign language? (Hint: First translate these percents into counts of the 500 students in the sample.)

12.27 Reaching dropouts. High school dropouts make up 13% of all Americans aged 18 to 24. A vocational school that wants to attract dropouts mails an advertising flyer to 25,000 persons between the ages of 18 and 24.

(a) If the mailing list can be considered a random sample of the population, what is the mean number of high school dropouts who will receive the flyer?

(b) What is the probability that at least 3500 dropouts will receive the flyer?

12.28 Is this coin balanced? While he was a prisoner of the Germans during World War II, John Kerrich tossed a coin 10,000 times. He got 5067 heads. Take Kerrich’s tosses to be an SRS from the population of all possible tosses of his coin. If the coin is perfectly balanced, \( p = 0.5 \). Is there reason to think that Kerrich’s coin gave too many heads to be balanced? To answer this question, find the probability that a balanced coin would give 5067 or more heads in 10,000 tosses. What do you conclude?

12.29 Inspecting switches. Example 12.5 concerns the count of bad switches in inspection samples of size 10. The count has the binomial distribution with \( n = 10 \) and \( p = 0.1 \). Set these values for the number of tosses and probability of heads in the Probability applet. The example calculates that the probability of getting a sample with exactly 1 bad switch is 0.3874. Of course, when we inspect only a few lots, the proportion of samples with exactly 1 bad switch will differ from this probability. Click “Toss” and “Reset” repeatedly to simulate inspecting 20 lots. Record the number of bad switches (the count of heads) in each of the 20 samples. What proportion of the 20 lots had exactly 1 bad switch? Remember that probability tells us only what happens in the long run.