In this chapter we cover...

- The idea of probability
- Probability models
- Probability rules
- Discrete probability models
- Continuous probability models
- Random variables
- Personal probability

Introducing Probability

Why is probability, the mathematics of chance behavior, needed to understand statistics, the science of data? Let’s look at a typical sample survey.

**Example 10.1 Do you lotto?**

What proportion of all adults bought a lottery ticket in the past 12 months? We don’t know, but we do have results from the Gallup Poll. Gallup took a random sample of 1523 adults. The poll found that 868 of the people in the sample bought tickets. The proportion who bought tickets was

\[
sample \ proportion = \frac{868}{1523} = 0.57 \quad (\text{that is, 57%})
\]

Because all adults had the same chance to be among the chosen 1523, it seems reasonable to use this 57% as an estimate of the unknown proportion in the population. It’s a fact that 57% of the sample bought lottery tickets—we know because Gallup asked them. We don’t know what percent of all adults bought tickets, but we estimate that about 57% did. This is a basic move in statistics: use a result from a sample to estimate something about a population.

What if Gallup took a second random sample of 1523 adults? The new sample would have different people in it. It is almost certain that there would not be exactly 868 positive responses. That is, Gallup’s estimate of the proportion of adults who bought a lottery ticket will vary from sample to sample. Could it happen that one random sample finds that 57% of adults recently bought a lottery ticket and a second random sample finds that only 37% had done so? Random samples eliminate
bias from the act of choosing a sample, but they can still be wrong because of the variability that results when we choose at random. If the variation when we take repeat samples from the same population is too great, we can’t trust the results of any one sample.

This is where we need facts about probability to make progress in statistics. Because Gallup uses chance to choose its samples, the laws of probability govern the behavior of the samples. Gallup says that the probability is 0.95 that an estimate from one of their samples comes within ±3 percentage points of the truth about the population of all adults. The first step toward understanding this statement is to understand what “probability 0.95” means. Our purpose in this chapter is to understand the language of probability, but without going into the mathematics of probability theory.

The idea of probability

To understand why we can trust random samples and randomized comparative experiments, we must look closely at chance behavior. The big fact that emerges is this: chance behavior is unpredictable in the short run but has a regular and predictable pattern in the long run.

Toss a coin, or choose an SRS. The result can’t be predicted in advance, because the result will vary when you toss the coin or choose the sample repeatedly. But there is still a regular pattern in the results, a pattern that emerges clearly only after many repetitions. This remarkable fact is the basis for the idea of probability.

EXAMPLE 10.2 Coin tossing

When you toss a coin, there are only two possible outcomes, heads or tails. Figure 10.1 shows the results of tossing a coin 5000 times twice. For each number of tosses from 1 to 5000, we have plotted the proportion of those tosses that gave a head. Trial A (solid blue line) begins tail, head, tail, tail. You can see that the proportion of heads for Trial A starts at 0 on the first toss, rises to 0.5 when the second toss gives a head, then falls to 0.33 and 0.25 as we get two more tails. Trial B, on the other hand, starts with five straight heads, so the proportion of heads is 1 until the sixth toss.

The proportion of tosses that produce heads is quite variable at first. Trial A starts low and Trial B starts high. As we make more and more tosses, however, the proportion of heads for both trials gets close to 0.5 and stays there. If we made yet a third trial at tossing the coin a great many times, the proportion of heads would again settle down to 0.5 in the long run. This is the intuitive idea of probability. Probability 0.5 means “occurs half the time in a very large number of trials.” The probability 0.5 appears as a horizontal line on the graph.

We might suspect that a coin has probability 0.5 of coming up heads just because the coin has two sides. But we can’t be sure. The coin might be unbalanced. In fact, spinning a penny or nickel on a flat surface, rather than tossing the coin, doesn’t give heads probability 0.5. The idea of probability is empirical. That is, it is based on observation rather than theorizing. Probability describes what happens in very many trials, and we must actually observe many trials to pin down a
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The proportion of tosses of a coin that give a head changes as we make more tosses. Eventually, however, the proportion approaches 0.5, the probability of a head. This figure shows the results of two trials of 5000 tosses each.

**EXAMPLE 10.3 Some coin throwers**

The French naturalist Count Buffon (1707–1788) tossed a coin 4040 times. Result: 2048 heads, or proportion \( \frac{2048}{4040} = 0.5069 \) for heads.

Around 1900, the English statistician Karl Pearson heroically tossed a coin 24,000 times. Result: 12,012 heads, a proportion of 0.5005.

While imprisoned by the Germans during World War II, the South African mathematician John Kerrich tossed a coin 10,000 times. Result: 5067 heads, a proportion of 0.5067.

**RANDOMNESS AND PROBABILITY**

We call a phenomenon **random** if individual outcomes are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.

The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

That some things are random is an observed fact about the world. The outcome of a coin toss, the time between emissions of particles by a radioactive source, and
The idea of probability

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the sexes of the next litter of lab rats are all random. So is the outcome of a random sample or a randomized experiment. Probability theory is the branch of mathematics that describes random behavior. Of course, we can never observe a probability exactly. We could always continue tossing the coin, for example. Mathematical probability is an idealization based on imagining what would happen in an indefinitely long series of trials.

The best way to understand randomness is to observe random behavior, as in Figure 10.1. You can do this with physical devices like coins, but computer simulations (imitations) of random behavior allow faster exploration. The Probability applet is a computer simulation that animates Figure 10.1. It allows you to choose the probability of a head and simulate any number of tosses of a coin with that probability. Experience shows that the proportion of heads gradually settles down close to the probability. Equally important, it also shows that the proportion in a small or moderate number of tosses can be far from the probability. Probability describes only what happens in the long run.

Computer simulations like the Probability applet start with given probabilities and imitate random behavior, but we can estimate a real-world probability only by actually observing many trials. Nonetheless, computer simulations are very useful because we need long runs of trials. In situations such as coin tossing, the proportion of an outcome often requires several hundred trials to settle down to the probability of that outcome. Short runs give only rough estimates of a probability.

**APPLY YOUR KNOWLEDGE**

10.1 **Texas Hold’em.** In the popular Texas Hold’em variety of poker, players make their best five-card poker hand by combining the two cards they are dealt with three of five cards available to all players. You read in a book on poker that if you hold a pair (two cards of the same rank) in your hand, the probability of getting four of a kind is 88/1000. Explain carefully what this means. In particular, explain why it does not mean that if you play 1000 such hands, exactly 88 will be four of a kind.

10.2 **Tossing a thumbtack.** Toss a thumbtack on a hard surface 100 times. How many times did it land with the point up? What is the approximate probability of landing point up?

10.3 **Random digits.** The table of random digits (Table B) was produced by a random mechanism that gives each digit probability 0.1 of being a 0.

(a) What proportion of the first 50 digits in the table are 0s? This proportion is an estimate, based on 50 repetitions, of the true probability, which in this case is known to be 0.1.

(b) The Probability applet can imitate random digits. Set the probability of heads in the applet to 0.1. Check “Show true probability” to show this value on the graph. A head stands for a 0 in the random digit table and a tail stands for any other digit. Simulate 200 digits (40 at a time—don’t click “Reset”). If you kept going forever, presumably you would get 10% heads. What was the result of your 200 tosses?
10.4 Probability says . . . Probability is a measure of how likely an event is to occur. Match one of the probabilities that follow with each statement of likelihood given. (The probability is usually a more exact measure of likelihood than is the verbal statement.)

\[
0 \quad 0.01 \quad 0.3 \quad 0.6 \quad 0.99 \quad 1
\]

(a) This event is impossible. It can never occur.
(b) This event is certain. It will occur on every trial.
(c) This event is very unlikely, but it will occur once in a while in a long sequence of trials.
(d) This event will occur more often than not.

**Probability models**

Gamblers have known for centuries that the fall of coins, cards, and dice displays clear patterns in the long run. The idea of probability rests on the observed fact that the average result of many thousands of chance outcomes can be known with near certainty. How can we give a mathematical description of long-run regularity?

To see how to proceed, think first about a very simple random phenomenon, tossing a coin once. When we toss a coin, we cannot know the outcome in advance. What do we know? We are willing to say that the outcome will be either heads or tails. We believe that each of these outcomes has probability 1/2. This description of coin tossing has two parts:

- A list of possible outcomes.
- A probability for each outcome.

Such a description is the basis for all probability models. Here is the basic vocabulary we use.

**PROBABILITY MODELS**

The **sample space** \( S \) of a random phenomenon is the set of all possible outcomes.

An **event** is an outcome or a set of outcomes of a random phenomenon. That is, an event is a subset of the sample space.

A **probability model** is a mathematical description of a random phenomenon consisting of two parts: a sample space \( S \) and a way of assigning probabilities to events.
A sample space \( S \) can be very simple or very complex. When we toss a coin once, there are only two outcomes, heads and tails. The sample space is \( S = \{H, T\} \). When Gallup draws a random sample of 1523 adults, the sample space contains all possible choices of 1523 of the 225 million adults in the country. This \( S \) is extremely large. Each member of \( S \) is a possible sample, which explains the term sample space.

**Example 10.4 Rolling dice**

Rolling two dice is a common way to lose money in casinos. There are 36 possible outcomes when we roll two dice and record the up-faces in order (first die, second die). Figure 10.2 displays these outcomes. They make up the sample space \( S \). “Roll a 5” is an event, call it \( A \), that contains four of these 36 outcomes:

\[
A = \{1\,\square\, \square, \ 1\,\square, \ 1\,\square, \ 1\,\square\, \square\, \square\, \square\, \square\, \square\} 
\]

How can we assign probabilities to this sample space? We can find the actual probabilities for two specific dice only by actually tossing the dice many times, and even then only approximately. So we will give a probability model that assumes ideal, perfectly balanced dice. This model will be quite accurate for carefully made casino dice and less accurate for the cheap dice that come with a board game.

If the dice are perfectly balanced, all 36 outcomes in Figure 10.2 will be equally likely. That is, each of the 36 outcomes will come up on one thirty-sixth of all rolls in the long run. So each outcome has probability \( 1/36 \). There are 4 outcomes in the event \( A \) (“roll a 5”), so this event has probability \( 4/36 \). In this way we can assign a probability to any event. So we have a complete probability model.

**Figure 10.2** The 36 possible outcomes in rolling two dice. If the dice are carefully made, all of these outcomes have the same probability.

**Example 10.5 Rolling dice and counting the spots**

Gamblers care only about the total number of spots on the up-faces of the dice. The sample space for rolling two dice and counting the spots is

\[
S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} 
\]
Comparing this $S$ with Figure 10.2 reminds us that we can change $S$ by changing the detailed description of the random phenomenon we are describing.

What are the probabilities for this new sample space? The 11 possible outcomes are not equally likely, because there are six ways to roll a 7 and only one way to roll a 2 or a 12. That's the key: each outcome in Figure 10.2 has probability 1/36. So “roll a 7” has probability 6/36 because this event contains 6 of the 36 outcomes. Similarly, “roll a 2” has probability 1/36, and “roll a 5” (4 outcomes from Figure 10.2) has probability 4/36. Here is the complete probability model:

<table>
<thead>
<tr>
<th>Number of spots</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
</table>

**APPLY YOUR KNOWLEDGE**

10.5 Sample space. Choose a student at random from a large statistics class. Describe a sample space $S$ for each of the following. (In some cases you may have some freedom in specifying $S$.)

(a) Ask how much time the student spent studying during the past 24 hours.
(b) Ask how much money in coins (not bills) the student is carrying.
(c) Record the student’s letter grade at the end of the course.
(d) Ask whether the student did or did not take a math class in each of the two previous years of school.

10.6 Dungeons & Dragons. Role-playing games such as Dungeons & Dragons use many different types of dice. A four-sided die has faces with 1, 2, 3, and 4 spots.

(a) What is the sample space for rolling the die twice (spots on first and second rolls)? Follow the example of Figure 10.2.
(b) What is the assignment of probabilities to outcomes in this sample space? Assume that the die is perfectly balanced, and follow the method of Example 10.4.

10.7 Dungeons & Dragons. The intelligence of a character in the game is determined by rolling the four-sided die twice and adding 1 to the sum of the spots. Start with your work in the previous exercise to give a probability model (sample space and probabilities of outcomes) for the character’s intelligence. Follow the method of Example 10.5.

**Probability rules**

In Examples 10.4 and 10.5 we found probabilities for perfectly balanced dice. As random phenomena go, dice are pretty simple. Even so, we had to assume idealized dice rather than working with real dice. In most situations, it isn’t easy to give a “correct” probability model. We can make progress by listing some facts that must be true for any assignment of probabilities. These facts follow from the idea of probability as “the long-run proportion of repetitions on which an event occurs.”
1. Any probability is a number between 0 and 1. Any proportion is a number between 0 and 1, so any probability is also a number between 0 and 1. An event with probability 0 never occurs, and an event with probability 1 occurs on every trial. An event with probability 0.5 occurs in half the trials in the long run.

2. All possible outcomes together must have probability 1. Because some outcome must occur on every trial, the sum of the probabilities for all possible outcomes must be exactly 1.

3. If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities. If one event occurs in 40% of all trials, a different event occurs in 25% of all trials, and the two can never occur together, then one or the other occurs on 65% of all trials because 40% + 25% = 65%.

4. The probability that an event does not occur is 1 minus the probability that the event does occur. If an event occurs in (say) 70% of all trials, it fails to occur in the other 30%. The probability that an event occurs and the probability that it does not occur always add to 100%, or 1.

We can use mathematical notation to state Facts 1 to 4 more concisely. Capital letters near the beginning of the alphabet denote events. If $A$ is any event, we write its probability as $P(A)$. Here are our probability facts in formal language. As you apply these rules, remember that they are just another form of intuitively true facts about long-run proportions.

**PROBABILITY RULES**

**Rule 1.** The probability $P(A)$ of any event $A$ satisfies $0 \leq P(A) \leq 1$.

**Rule 2.** If $S$ is the sample space in a probability model, then $P(S) = 1$.

**Rule 3.** Two events $A$ and $B$ are disjoint if they have no outcomes in common and so can never occur together. If $A$ and $B$ are disjoint,

$$P(A \text{ or } B) = P(A) + P(B)$$

This is the addition rule for disjoint events.

**Rule 4.** For any event $A$,

$$P(A \text{ does not occur}) = 1 - P(A)$$

The addition rule extends to more than two events that are disjoint in the sense that no two have any outcomes in common. If events $A$, $B$, and $C$ are disjoint, the probability that one of these events occurs is $P(A) + P(B) + P(C)$. 

**Equally likely?**

A game of bridge begins by dealing all 52 cards in the deck to the four players, 13 to each. If the deck is well shuffled, all of the immense number of possible hands will be equally likely. But don’t expect the hands that appear in newspaper bridge columns to reflect the equally likely probability model. Writers on bridge choose “interesting” hands, especially those that lead to high bids that are rare in actual play.
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Example 10.6 Using the probability rules

We already used the addition rule, without calling it by that name, to find the probabilities in Example 10.5. The event “roll a 5” contains the four disjoint outcomes displayed in Example 10.4, so the addition rule (Rule 3) says that its probability is

\[ P(\text{roll a 5}) = P(\text{1}) + P(\text{3}) + P(\text{4}) + P(\text{6}) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{4}{36} = 0.111 \]

Check that the probabilities in Example 10.5, found using the addition rule, are all between 0 and 1 and add to exactly 1. That is, this probability model obeys Rules 1 and 2.

What is the probability of rolling anything other than a 5? By Rule 4,

\[ P(\text{roll does not give a 5}) = 1 - P(\text{roll a 5}) = 1 - 0.111 = 0.889 \]

Our model assigns probabilities to individual outcomes. To find the probability of an event, just add the probabilities of the outcomes that make up the event. For example:

\[ P(\text{outcome is odd}) = P(3) + P(5) + P(7) + P(9) + P(11) = \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \frac{18}{36} = \frac{1}{2} \]

Apply Your Knowledge

10.8 Preparing for the GMAT. A company that offers courses to prepare students for the Graduate Management Admission test (GMAT) has the following information about its customers: 20% are currently undergraduate students in business; 15% are undergraduate students in other fields of study; 60% are college graduates who are currently employed; and 5% are college graduates who are not employed.

(a) Does this assignment of probabilities to customer backgrounds satisfy Rules 1 and 2?

(b) What percent of customers are currently undergraduates?

10.9 Languages in Canada. Canada has two official languages, English and French. Choose a Canadian at random and ask, “What is your mother tongue?” Here is the distribution of responses, combining many separate languages from the broad Asian/Pacific region:

<table>
<thead>
<tr>
<th>Language</th>
<th>English</th>
<th>French</th>
<th>Asian/Pacific</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.59</td>
<td>0.23</td>
<td>0.07</td>
<td>?</td>
</tr>
</tbody>
</table>

(a) What probability should replace “?” in the distribution?

(b) What is the probability that a Canadian’s mother tongue is not English?
Discrete probability models

Examples 10.4, 10.5, and 10.6 illustrate one way to assign probabilities to events: assign a probability to every individual outcome, then add these probabilities to find the probability of any event. This idea works well when there are only a finite (fixed and limited) number of outcomes.

**DISCRETE PROBABILITY MODEL**

A probability model with a finite sample space is called **discrete**.

To assign probabilities in a discrete model, list the probabilities of all the individual outcomes. These probabilities must be numbers between 0 and 1 and must have sum 1. The probability of any event is the sum of the probabilities of the outcomes making up the event.

**EXAMPLE 10.7  Benford's law**

Faked numbers in tax returns, invoices, or expense account claims often display patterns that aren’t present in legitimate records. Some patterns, like too many round numbers, are obvious and easily avoided by a clever crook. Others are more subtle. It is a striking fact that the first digits of numbers in legitimate records often follow a model known as Benford’s law.\(^2\) Call the first digit of a randomly chosen record \(X\) for short. Benford’s law gives this probability model for \(X\) (note that a first digit can’t be 0):

<table>
<thead>
<tr>
<th>First digit (X)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.301</td>
<td>0.176</td>
<td>0.125</td>
<td>0.097</td>
<td>0.079</td>
<td>0.067</td>
<td>0.058</td>
<td>0.051</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Check that the probabilities of the outcomes sum to exactly 1. This is therefore a legitimate discrete probability model. Investigators can detect fraud by comparing the first digits in records such as invoices paid by a business with these probabilities.

The probability that a first digit is equal to or greater than 6 is

\[
P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) = 0.067 + 0.058 + 0.051 + 0.046 = 0.222
\]

This is less than the probability that a record has first digit 1,

\[
P(X = 1) = 0.301
\]

Fraudulent records tend to have too few 1s and too many higher first digits.

**CAUTION**

Note that the probability that a first digit is greater than or equal to 6 is not the same as the probability that a first digit is strictly greater than 6. The latter probability is

\[
P(X > 6) = 0.058 + 0.051 + 0.046 = 0.155
\]

The outcome \(X = 6\) is included in “greater than or equal to” and is not included in “strictly greater than.”
APPLY YOUR KNOWLEDGE

10.10 Rolling a die. Figure 10.3 displays several discrete probability models for rolling a die. We can learn which model is actually accurate for a particular die only by rolling the die many times. However, some of the models are not legitimate. That is, they do not obey the rules. Which are legitimate and which are not? In the case of the illegitimate models, explain what is wrong.

![Figure 10.3](image)

**FIGURE 10.3** Four assignments of probabilities to the six faces of a die, for Exercise 10.10.

10.11 Benford’s law. The first digit of a randomly chosen expense account claim follows Benford’s law (Example 10.7). Consider the events

\[ A = \{ \text{first digit is 7 or greater} \} \quad B = \{ \text{first digit is odd} \} \]

(a) What outcomes make up the event \( A \)? What is \( P(A) \)?
(b) What outcomes make up the event \( B \)? What is \( P(B) \)?
(c) What outcomes make up the event “\( A \text{ or } B \)”? What is \( P(A \text{ or } B) \)? Why is this probability not equal to \( P(A) + P(B) \)?

10.12 Watching TV. Choose a young person (age 19 to 25) at random and ask, “In the past seven days, how many days did you watch television?” Call the response \( X \) for short. Here is a probability model for the response:

<table>
<thead>
<tr>
<th>Days</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>1</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.06</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>0.57</td>
</tr>
</tbody>
</table>

(a) Verify that this is a legitimate discrete probability model.
(b) Describe the event \( X < 7 \) in words. What is \( P(X < 7) \)?
(c) Express the event “watched TV at least once” in terms of \( X \). What is the probability of this event?
Continuous probability models

When we use the table of random digits to select a digit between 0 and 9, the discrete probability model assigns probability $1/10$ to each of the 10 possible outcomes. Suppose that we want to choose a number at random between 0 and 1, allowing any number between 0 and 1 as the outcome. Software random number generators will do this. The sample space is now an entire interval of numbers:

$$S = \{\text{all numbers between 0 and 1}\}$$

Call the outcome of the random number generator $Y$ for short. How can we assign probabilities to such events as $\{0.3 \leq Y \leq 0.7\}$? As in the case of selecting a random digit, we would like all possible outcomes to be equally likely. But we cannot assign probabilities to each individual value of $Y$ and then add them, because there are infinitely many possible values.

We use a new way of assigning probabilities directly to events—as areas under a density curve. Any density curve has area exactly 1 underneath it, corresponding to total probability 1. We first met density curves as models for data in Chapter 3 (page 64).

**CONTINUOUS PROBABILITY MODEL**

A continuous probability model assigns probabilities as areas under a density curve. The area under the curve and above any range of values is the probability of an outcome in that range.

**EXAMPLE 10.8 Random numbers**

The random number generator will spread its output uniformly across the entire interval from 0 to 1 as we allow it to generate a long sequence of numbers. The results of many trials are represented by the uniform density curve shown in Figure 10.4. This density curve has height 1 over the interval from 0 to 1. The area under the curve is 1, and the probability of any event is the area under the curve and above the event in question.

As Figure 10.4(a) illustrates, the probability that the random number generator produces a number between 0.3 and 0.7 is

$$P(0.3 \leq Y \leq 0.7) = 0.4$$

because the area under the density curve and above the interval from 0.3 to 0.7 is 0.4. The height of the curve is 1 and the area of a rectangle is the product of height and length, so the probability of any interval of outcomes is just the length of the interval. Similarly,

$$P(Y \leq 0.5) = 0.5$$

$$P(Y > 0.8) = 0.2$$

$$P(Y \leq 0.5 \text{ or } Y > 0.8) = 0.7$$
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The last event consists of two nonoverlapping intervals, so the total area above the event is found by adding two areas, as illustrated by Figure 10.4(b). This assignment of probabilities obeys all of our rules for probability.

The probability model for a continuous random variable assigns probabilities to intervals of outcomes rather than to individual outcomes. In fact, all continuous probability models assign probability 0 to every individual outcome. Only intervals of values have positive probability. To see that this is true, consider a specific outcome such as $P(Y = 0.8)$ in Example 10.8. The probability of any interval is the same as its length. The point 0.8 has no length, so its probability is 0.

We can use any density curve to assign probabilities. The density curves that are most familiar to us are the Normal curves. So Normal distributions are probability models. There is a close connection between a Normal distribution as an idealized description for data and a Normal probability model. If we look at the heights of all young women, we find that they closely follow the Normal distribution with mean $\mu = 64$ inches and standard deviation $\sigma = 2.7$ inches. That is a distribution for a large set of data. Now choose one young woman at random. Call her height $X$. If we repeat the random choice very many times, the distribution of values of $X$ is the same Normal distribution.

**EXAMPLE 10.9 The heights of young women**

What is the probability that a randomly chosen young woman has height between 68 and 70 inches? The height $X$ of the woman we choose has the $N(64, 2.7)$ distribution. We want $P(68 \leq X \leq 70)$. Software or the Normal Curve applet will give us the answer at once.

We can also find the probability by standardizing and using Table A, the table of standard Normal probabilities. We will reserve capital $Z$ for a standard Normal variable.

$$P(68 \leq X \leq 70) = P\left( \frac{68 - 64}{2.7} \leq \frac{X - 64}{2.7} \leq \frac{70 - 64}{2.7} \right) = P(1.48 \leq Z \leq 2.22) = 0.9868 - 0.9306 = 0.0562$$
Standard Normal curve Probability = 0.0562

Figure 10.5 shows the area under the standard Normal curve. The calculation is the same as those we did in Chapter 3. Only the language of probability is new.

APPLY YOUR KNOWLEDGE

10.13 Random numbers. Let $X$ be a random number between 0 and 1 produced by the idealized random number generator described in Example 10.8 and Figure 10.4. Find the following probabilities:

(a) $P(X \leq 0.4)$
(b) $P(X < 0.4)$
(c) $P(0.3 \leq X \leq 0.5)$

10.14 Adding random numbers. Generate two random numbers between 0 and 1 and take $Y$ to be their sum. The sum $Y$ can take any value between 0 and 2. The density curve of $Y$ is the triangle shown in Figure 10.6.

(a) Verify by geometry that the area under this curve is 1.

Figure 10.6 The density curve for the sum of two random numbers, for Exercise 10.14. This density curve spreads probability between 0 and 2.
(b) What is the probability that $Y$ is less than 1? (Sketch the density curve, shade the area that represents the probability, then find that area. Do this for (c) also.)

(c) What is the probability that $Y$ is less than 0.5?

10.15 Iowa Test scores. The Normal distribution with mean $\mu = 6.8$ and standard deviation $\sigma = 1.6$ is a good description of the Iowa Test vocabulary scores of seventh-grade students in Gary, Indiana. This is a continuous probability model for the score of a randomly chosen student. Figure 3.1 (page 65) pictures the density curve. Call the score of a randomly chosen student $X$ for short.

(a) Write the event “the student chosen has a score of 10 or higher” in terms of $X$.

(b) Find the probability of this event.

Random variables

Examples 10.7 to 10.9 use a shorthand notation that is often convenient. In Example 10.9, we let $X$ stand for the result of choosing a woman at random and measuring her height. We know that $X$ would take a different value if we made another random choice. Because its value changes from one random choice to another, we call the height $X$ a random variable.

**RANDOM VARIABLE**

A random variable is a variable whose value is a numerical outcome of a random phenomenon.

The probability distribution of a random variable $X$ tells us what values $X$ can take and how to assign probabilities to those values.

We usually denote random variables by capital letters near the end of the alphabet, such as $X$ or $Y$. Of course, the random variables of greatest interest to us are outcomes such as the mean $\bar{x}$ of a random sample, for which we will keep the familiar notation. There are two main types of random variables, corresponding to two types of probability models: discrete and continuous.

**Example 10.10 Discrete and continuous random variables**

The first digit $X$ in Example 10.7 is a random variable whose possible values are the whole numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The distribution of $X$ assigns a probability to each of these outcomes. Random variables that have a finite list of possible outcomes are called discrete.

Compare the output $Y$ of the random number generator in Example 10.8. The values of $Y$ fill the entire interval of numbers between 0 and 1. The probability distribution of $Y$ is given by its density curve, shown in Figure 10.4. Random variables that can take on any value in an interval, with probabilities given as areas under a density curve, are called continuous.
10.16 Grades in a statistics course. North Carolina State University posts the grade distributions for its courses online. Students in Statistics 302 in the Spring 2005 semester received 45% A's, 35% B's, 16% C's, 2% D's, and 2% F's. Choose a Statistics 302 student at random. To “choose at random” means to give every student the same chance to be chosen. The student’s grade on a four-point scale (with A = 4) is a discrete random variable X with this probability distribution:

<table>
<thead>
<tr>
<th>Value of X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.02</td>
<td>0.02</td>
<td>0.16</td>
<td>0.35</td>
<td>0.45</td>
</tr>
</tbody>
</table>

(a) Say in words what the meaning of $P(X \geq 3)$ is. What is this probability?
(b) Write the event “the student got a grade poorer than C” in terms of values of the random variable X. What is the probability of this event?

10.17 ACT scores. ACT scores for the 1,171,460 members of the 2004 high school graduating class who took the test closely follow the Normal distribution with mean 20.9 and standard deviation 4.8. Choose a student at random from this group and let $Y$ be his or her ACT score. Write the event “the student’s score was higher than 25” in terms of $Y$ and find its probability.

**Personal probability**

We began our discussion of probability with one idea: the probability of an outcome of a random phenomenon is the proportion of times that outcome would occur in a very long series of repetitions. This idea ties probability to actual outcomes. It allows us, for example, to estimate probabilities by simulating random phenomena. Yet we often meet another, quite different, idea of probability.

**Example 10.11 Joe and the Chicago Cubs**

Joe sits staring into his beer as his favorite baseball team, the Chicago Cubs, loses another game. The Cubbies have some good young players, so let’s ask Joe, “What’s the chance that the Cubs will go to the World Series next year?” Joe brightens up. “Oh, about 10%,” he says.

Does Joe assign probability 0.10 to the Cubs’ appearing in the World Series? The outcome of next year’s pennant race is certainly unpredictable, but we can’t reasonably ask what would happen in many repetitions. Next year’s baseball season will happen only once and will differ from all other seasons in players, weather, and many other ways. If probability measures “what would happen if we did this many times,” Joe’s 0.10 is not a probability. Probability is based on data about many repetitions of the same random phenomenon. Joe is giving us something else, his personal judgment.

**What are the odds?**

Gamblers often express chance in terms of odds rather than probability. Odds of $A$ to $B$ against an outcome means that the probability of that outcome is $B/(A+B)$. So “odds of 5 to 1” is another way of saying “probability 1/6.” A probability is always between 0 and 1, but odds range from 0 to infinity. Although odds are mainly used in gambling, they give us a way to make very small probabilities clearer. “Odds of 999 to 1” may be easier to understand than “probability 0.001.”

*This section is optional.*
Although Joe’s 0.10 isn’t a probability in our usual sense, it gives useful information about Joe’s opinion. More seriously, a company asking, “How likely is it that building this plant will pay off within five years?” can’t employ an idea of probability based on many repetitions of the same thing. The opinions of company officers and advisers are nonetheless useful information, and these opinions can be expressed in the language of probability. These are personal probabilities.

**PERSONAL PROBABILITY**

A personal probability of an outcome is a number between 0 and 1 that expresses an individual’s judgment of how likely the outcome is.

Rachel’s opinion about the Cubs may differ from Joe’s, and the opinions of several company officers about the new plant may differ. Personal probabilities are indeed personal: they vary from person to person. Moreover, a personal probability can’t be called right or wrong. If we say, “In the long run, this coin will come up heads 60% of the time,” we can find out if we are right by actually tossing the coin several thousand times. If Joe says, “I think the Cubs have a 10% chance of going to the World Series next year,” that’s just Joe’s opinion. Why think of personal probabilities as probabilities? Because any set of personal probabilities that makes sense obeys the same basic Rules 1 to 4 that describe any legitimate assignment of probabilities to events. If Joe thinks there’s a 10% chance that the Cubs will go to the World Series, he must also think that there’s a 90% chance that they won’t go. There is just one set of rules of probability, even though we now have two interpretations of what probability means.

**APPLY YOUR KNOWLEDGE**

10.18 Will you have an accident? The probability that a randomly chosen driver will be involved in an accident in the next year is about 0.2. This is based on the proportion of millions of drivers who have accidents. “Accident” includes things like crumpling a fender in your own driveway, not just highway accidents.

(a) What do you think is your own probability of being in an accident in the next year? This is a personal probability.

(b) Give some reasons why your personal probability might be a more accurate prediction of your “true chance” of having an accident than the probability for a random driver.

(c) Almost everyone says their personal probability is lower than the random driver probability. Why do you think this is true?

**CHAPTER 10 SUMMARY**

A random phenomenon has outcomes that we cannot predict but that nonetheless have a regular distribution in very many repetitions.
The **probability** of an event is the proportion of times the event occurs in many repeated trials of a random phenomenon.

A **probability model** for a random phenomenon consists of a sample space \( S \) and an assignment of probabilities \( P \).

The **sample space** \( S \) is the set of all possible outcomes of the random phenomenon. Sets of outcomes are called **events**. \( P \) assigns a number \( P(A) \) to an event \( A \) as its probability.

Any assignment of probability must obey the rules that state the basic properties of probability:

1. \( 0 \leq P(A) \leq 1 \) for any event \( A \).
2. \( P(S) = 1 \).
3. **Addition rule**: Events \( A \) and \( B \) are **disjoint** if they have no outcomes in common. If \( A \) and \( B \) are disjoint, then \( P(A \text{ or } B) = P(A) + P(B) \).
4. For any event \( A \), \( P(A \text{ does not occur}) = 1 - P(A) \).

When a sample space \( S \) contains finitely many possible values, a **discrete probability model** assigns each of these values a probability between 0 and 1 such that the sum of all the probabilities is exactly 1. The probability of any event is the sum of the probabilities of all the values that make up the event.

A sample space can contain all values in some interval of numbers. A **continuous probability model** assigns probabilities as areas under a density curve. The probability of any event is the area under the curve above the values that make up the event.

A **random variable** is a variable taking numerical values determined by the outcome of a random phenomenon. The **probability distribution** of a random variable \( X \) tells us what the possible values of \( X \) are and how probabilities are assigned to those values.

A random variable \( X \) and its distribution can be **discrete** or **continuous**. A **discrete random variable** has finitely many possible values. Its distribution gives the probability of each value. A **continuous random variable** takes all values in some interval of numbers. A density curve describes the probability distribution of a continuous random variable.

**CHECK YOUR SKILLS**

10.19 You read in a book on poker that the probability of being dealt three of a kind in a five-card poker hand is 1/50. This means that

(a) if you deal thousands of poker hands, the fraction of them that contain three of a kind will be very close to 1/50.

(b) if you deal 50 poker hands, exactly 1 of them will contain three of a kind.

(c) if you deal 10,000 poker hands, exactly 200 of them will contain three of a kind.
### 10.20 A basketball player shoots 8 free throws during a game. The sample space for counting the number she makes is

(a) \( S = \) any number between 0 and 1.
(b) \( S = \) whole numbers 0 to 8.
(c) \( S = \) all sequences of 8 hits or misses, like HMMHHHMH.

Here is the probability model for the blood type of a randomly chosen person in the United States. Exercises 10.21 to 10.24 use this information.

<table>
<thead>
<tr>
<th>Blood type</th>
<th>O</th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.45</td>
<td>0.40</td>
<td>0.11</td>
<td>?</td>
</tr>
</tbody>
</table>

### 10.21 This probability model is

(a) continuous.  (b) discrete.  (c) equally likely.

### 10.22 The probability that a randomly chosen American has type AB blood must be

(a) any number between 0 and 1.  (b) 0.04.  (c) 0.4.

### 10.23 Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen American can donate blood to Maria?

(a) 0.11  (b) 0.44  (c) 0.56

### 10.24 What is the probability that a randomly chosen American does not have type O blood?

(a) 0.55  (b) 0.45  (c) 0.04

### 10.25 In a table of random digits such as Table B, each digit is equally likely to be any of 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. What is the probability that a digit in the table is a 0?

(a) 1/9  (b) 1/10  (c) 9/10

### 10.26 In a table of random digits such as Table B, each digit is equally likely to be any of 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. What is the probability that a digit in the table is 7 or greater?

(a) 7/10  (b) 4/10  (c) 3/10

### 10.27 Choose an American household at random and let the random variable \( X \) be the number of cars (including SUVs and light trucks) they own. Here is the probability model if we ignore the few households that own more than 5 cars:

<table>
<thead>
<tr>
<th>Number of cars ( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.09</td>
<td>0.36</td>
<td>0.35</td>
<td>0.13</td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?

(a) 20%  (b) 45%  (c) 55%

### 10.28 Choose a person at random and give him or her an IQ test. The result is a random variable \( Y \). The probability distribution of \( Y \) is the Normal distribution with mean \( \mu = 100 \) and standard deviation \( \sigma = 15 \). The probability \( P(Y > 120) \) that the person chosen has IQ score higher than 120 is about

(a) 0.908  (b) 0.184  (c) 0.092.
CHAPTER 10 EXERCISES

10.29 Nickels falling over. You may feel that it is obvious that the probability of a head in tossing a coin is about 1/2 because the coin has two faces. Such opinions are not always correct. Stand a nickel on edge on a hard, flat surface. Pound the surface with your hand so that the nickel falls over. What is the probability that it falls with heads upward? Make at least 50 trials to estimate the probability of a head.

10.30 Sample space. In each of the following situations, describe a sample space $S$ for the random phenomenon.
(a) A basketball player shoots four free throws. You record the sequence of hits and misses.
(b) A basketball player shoots four free throws. You record the number of baskets she makes.

10.31 Probability models? In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. If not, give specific reasons for your answer.
(a) Roll a die and record the count of spots on the up-face: $P(1) = 0$, $P(2) = 1/6$, $P(3) = 1/3$, $P(4) = 1/3$, $P(5) = 1/6$, $P(6) = 0$.
(b) Choose a college student at random and record sex and enrollment status: $P$(female full-time) = 0.56, $P$(female part-time) = 0.24, $P$(male full-time) = 0.44, $P$(male part-time) = 0.17.

10.32 Education among young adults. Choose a young adult (age 25 to 34) at random. The probability is 0.12 that the person chosen did not complete high school, 0.31 that the person has a high school diploma but no further education, and 0.29 that the person has at least a bachelor's degree.
(a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor's degree?
(b) What is the probability that a randomly chosen young adult has at least a high school education?

10.33 Deaths on the job. Government data on job-related deaths assign a single occupation to each such death that occurs in the United States. The data show that the probability is 0.134 that a randomly chosen death was agriculture-related, and 0.119 that it was manufacturing-related. What is the probability that a death was either agriculture-related or manufacturing-related? What is the probability that the death was related to some other occupation?

10.34 Loaded dice. There are many ways to produce crooked dice. To load a die so that 6 comes up too often and 1 (which is opposite 6) comes up too seldom, add a bit of lead to the filling of the spot on the 1 face. If a die is loaded so that 6 comes up with probability 0.2 and the probabilities of the 2, 3, 4, and 5 faces are not affected, what is the assignment of probabilities to the six faces?

10.35 What probability doesn’t say. The idea of probability is that the proportion of heads in many tosses of a balanced coin eventually gets close to 0.5. But does the actual count of heads get close to one-half the number of tosses? Let's find out. Set the “Probability of heads” in the Probability applet to 0.5 and the number of tosses.
to 40. You can extend the number of tosses by clicking “Toss” again to get 40 more. Don’t click “Reset” during this exercise.

(a) After 40 tosses, what is the proportion of heads? What is the count of heads? What is the difference between the count of heads and 20 (one-half the number of tosses)?

(b) Keep going to 120 tosses. Again record the proportion and count of heads and the difference between the count and 60 (half the number of tosses).

(c) Keep going. Stop at 240 tosses and again at 480 tosses to record the same facts. Although it may take a long time, the laws of probability say that the proportion of heads will always get close to 0.5 and also that the difference between the count of heads and half the number of tosses will always grow without limit.

10.36 A door prize. A party host gives a door prize to one guest chosen at random. There are 48 men and 42 women at the party. What is the probability that the prize goes to a woman? Explain how you arrived at your answer.

10.37 Land in Canada. Canada’s national statistics agency, Statistics Canada, says that the land area of Canada is 9,094,000 square kilometers. Of this land, 4,176,000 square kilometers are forested. Choose a square kilometer of land in Canada at random.

(a) What is the probability that the area you choose is forested?

(b) What is the probability that it is not forested?

10.38 Foreign language study. Choose a student in grades 9 to 12 at random and ask if he or she is studying a language other than English. Here is the distribution of results:

<table>
<thead>
<tr>
<th>Language</th>
<th>Spanish</th>
<th>French</th>
<th>German</th>
<th>All others</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.26</td>
<td>0.09</td>
<td>0.03</td>
<td>0.03</td>
<td>0.59</td>
</tr>
</tbody>
</table>

(a) Explain why this is a legitimate probability model.

(b) What is the probability that a randomly chosen student is studying a language other than English?

(c) What is the probability that a randomly chosen student is studying French, German, or Spanish?

10.39 Car colors. Choose a new car or light truck at random and note its color. Here are the probabilities of the most popular colors for vehicles made in North America in 2005:

<table>
<thead>
<tr>
<th>Color</th>
<th>Silver</th>
<th>White</th>
<th>Gray</th>
<th>Blue</th>
<th>Black</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.18</td>
<td>0.17</td>
<td>0.15</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

(a) What is the probability that the vehicle you choose has any color other than the six listed?

(b) What is the probability that a randomly chosen vehicle is neither silver nor white?
Chapter 10 Exercises

10.40 Colors of M&M’s. If you draw an M&M candy at random from a bag of the candies, the candy you draw will have one of six colors. The probability of drawing each color depends on the proportion of each color among all candies made. Here is the distribution for milk chocolate M&M’s:

<table>
<thead>
<tr>
<th>Color</th>
<th>Yellow</th>
<th>Red</th>
<th>Orange</th>
<th>Brown</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.14</td>
<td>0.13</td>
<td>0.20</td>
<td>0.13</td>
<td>0.16</td>
<td>?</td>
</tr>
</tbody>
</table>

(a) What must be the probability of drawing a blue candy?
(b) What is the probability that you do not draw a brown candy?
(c) What is the probability that the candy you draw is either yellow, orange, or red?

10.41 More M&M’s. You can create your own custom blend of M&M’s, with 21 colors to choose from. Cindy chooses equal numbers of teal, aqua green, light blue, dark blue, and light purple. When you choose a candy at random from Cindy’s custom blend, what is the probability for each color?

10.42 Race and ethnicity. The 2000 census allowed each person to choose from a long list of races. That is, in the eyes of the Census Bureau, you belong to whatever race you say you belong to. “Hispanic/Latino” is a separate category; Hispanics may be of any race. If we choose a resident of the United States at random, the 2000 census gives these probabilities:

<table>
<thead>
<tr>
<th></th>
<th>Hispanic</th>
<th>Not Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian</td>
<td>0.000</td>
<td>0.036</td>
</tr>
<tr>
<td>Black</td>
<td>0.003</td>
<td>0.121</td>
</tr>
<tr>
<td>White</td>
<td>0.060</td>
<td>0.691</td>
</tr>
<tr>
<td>Other</td>
<td>0.062</td>
<td>0.027</td>
</tr>
</tbody>
</table>

(a) Verify that this is a legitimate assignment of probabilities.
(b) What is the probability that a randomly chosen American is Hispanic?
(c) Non-Hispanic whites are the historical majority in the United States. What is the probability that a randomly chosen American is not a member of this group?

10.43 Spelling errors. Spell-checking software catches “nonword errors” that result in a string of letters that is not a word, as when “the” is typed as “teh.” When undergraduates are asked to type a 250-word essay (without spell-checking), the number $X$ of nonword errors has the following distribution:

<table>
<thead>
<tr>
<th>Value of $X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Is the random variable $X$ discrete or continuous? Why?
(b) Write the event “at least one nonword error” in terms of $X$. What is the probability of this event?
(c) Describe the event $X \leq 2$ in words. What is its probability? What is the probability that $X < 2$?

10.44 First digits again. A crook who never heard of Benford’s law might choose the first digits of his faked invoices so that all of 1, 2, 3, 4, 5, 6, 7, 8, and 9 are equally likely. Call the first digit of a randomly chosen fake invoice $W$ for short.
(a) Write the probability distribution for the random variable $W$.
(b) Find $P(W \geq 6)$ and compare your result with the Benford’s law probability from Example 10.7.

10.45 Who goes to Paris? Abby, Deborah, Mei-Ling, Sam, and Roberto work in a firm’s public relations office. Their employer must choose two of them to attend a conference in Paris. To avoid unfairness, the choice will be made by drawing two names from a hat. (This is an SRS of size 2.)
(a) Write down all possible choices of two of the five names. This is the sample space.
(b) The random drawing makes all choices equally likely. What is the probability of each choice?
(c) What is the probability that Mei-Ling is chosen?
(d) What is the probability that neither of the two men (Sam and Roberto) is chosen?

10.46 Birth order. A couple plans to have three children. There are 8 possible arrangements of girls and boys. For example, GGB means the first two children are girls and the third child is a boy. All 8 arrangements are (approximately) equally likely.
(a) Write down all 8 arrangements of the sexes of three children. What is the probability of any one of these arrangements?
(b) Let $X$ be the number of girls the couple has. What is the probability that $X = 2$?
(c) Starting from your work in (a), find the distribution of $X$. That is, what values can $X$ take, and what are the probabilities for each value?

10.47 Unusual dice. Nonstandard dice can produce interesting distributions of outcomes. You have two balanced, six-sided dice. One is a standard die, with faces having 1, 2, 3, 4, 5, and 6 spots. The other die has three faces with 0 spots and three faces with 6 spots. Find the probability distribution for the total number of spots $Y$ on the up-faces when you roll these two dice. (Hint: Start with a picture like Figure 10.2 for the possible up-faces. Label the three 0 faces on the second die 0a, 0b, 0c in your picture, and similarly distinguish the three 6 faces.)

10.48 Random numbers. Many random number generators allow users to specify the range of the random numbers to be produced. Suppose that you specify that the random number $Y$ can take any value between 0 and 2. Then the density curve of the outcomes has constant height between 0 and 2, and height 0 elsewhere.
(a) Is the random variable $Y$ discrete or continuous? Why?
(b) What is the height of the density curve between 0 and 2? Draw a graph of the density curve.
(c) Use your graph from (b) and the fact that probability is area under the curve to find $P(Y \leq 1)$.
10.49 **Did you vote?** A sample survey contacted an SRS of 663 registered voters in Oregon shortly after an election and asked respondents whether they had voted. Voter records show that 56% of registered voters had actually voted. We will see later that in this situation the proportion of the sample who voted (call this proportion \( \hat{p} \)) has approximately the Normal distribution with mean \( \mu = 0.56 \) and standard deviation \( \sigma = 0.019 \).

(a) If the respondents answer truthfully, what is \( P(0.52 \leq \hat{p} \leq 0.60) \)? This is the probability that the sample proportion \( \hat{p} \) estimates the population proportion 0.56 within plus or minus 0.04.

(b) In fact, 72% of the respondents said they had voted (\( \hat{p} = 0.72 \)). If respondents answer truthfully, what is \( P(\hat{p} \geq 0.72) \)? This probability is so small that it is good evidence that some people who did not vote claimed that they did vote.

10.50 **More random numbers.** Find these probabilities as areas under the density curve you sketched in Exercise 10.48.

(a) \( P(0.5 < Y < 1.3) \).

(b) \( P(Y \geq 0.8) \).

10.51 **NAEP math scores.** Scores on the latest National Assessment of Educational Progress 12th-grade mathematics test were approximately Normal with mean 300 points (out of 500 possible) and standard deviation 35 points. Let \( Y \) stand for the score of a randomly chosen student. Express each of the following events in terms of \( Y \) and use the 68–95–99.7 rule to give the approximate probability.

(a) The student has a score above 300.

(b) The student’s score is above 370.

10.52 **Playing Pick 4.** The Pick 4 games in many state lotteries announce a four-digit winning number each day. The winning number is essentially a four-digit group from a table of random digits. You win if your choice matches the winning digits. Suppose your chosen number is 5974.

(a) What is the probability that your number matches the winning number exactly?

(b) What is the probability that your number matches the digits in the winning number in any order?

10.53 **Friends.** How many close friends do you have? Suppose that the number of close friends adults claim to have varies from person to person with mean \( \mu = 9 \) and standard deviation \( \sigma = 2.5 \). An opinion poll asks this question of an SRS of 1100 adults. We will see later that in this situation the sample mean response \( \bar{x} \) has approximately the Normal distribution with mean 9 and standard deviation 0.075. What is \( P(8.9 \leq \bar{x} \leq 9.1) \), the probability that the sample result \( \bar{x} \) estimates the population truth \( \mu = 9 \) to within \( \pm 0.1 \)?

10.54 **Playing Pick 4, continued.** The Wisconsin version of Pick 4 pays out $5000 on a $1 bet if your number matches the winning number exactly. It pays $200 on a $1 bet if the digits in your number match those of the winning number in any order. You choose which of these two bets to make. On the average over many bets, your winnings will be

\[
\text{mean amount won} = \text{payout amount} \times \text{probability of winning}
\]
What are the mean payout amounts for these two bets? Is one of the two bets a better choice?

10.55 Shaq's free throws. The basketball player Shaquille O'Neal makes about half of his free throws over an entire season. Use the Probability applet or software to simulate 100 free throws shot by a player who has probability 0.5 of making each shot. (In most software, the key phrase to look for is “Bernoulli trials.” This is the technical term for independent trials with Yes/No outcomes. Our outcomes here are “Hit” and “Miss.”)

(a) What percent of the 100 shots did he hit?
(b) Examine the sequence of hits and misses. How long was the longest run of shots made? Of shots missed? (Sequences of random outcomes often show runs longer than our intuition thinks likely.)

10.56 Simulating an opinion poll. A recent opinion poll showed that about 65% of the American public have a favorable opinion of the software company Microsoft. Suppose that this is exactly true. Choosing a person at random then has probability 0.65 of getting one who has a favorable opinion of Microsoft. Use the Probability applet or your statistical software to simulate choosing many people at random. (In most software, the key phrase to look for is “Bernoulli trials.” This is the technical term for independent trials with Yes/No outcomes. Our outcomes here are “Favorable” or not.)

(a) Simulate drawing 20 people, then 80 people, then 320 people. What proportion has a favorable opinion of Microsoft in each case? We expect (but because of chance variation we can’t be sure) that the proportion will be closer to 0.65 in longer runs of trials.
(b) Simulate drawing 20 people 10 times and record the percents in each sample who have a favorable opinion of Microsoft. Then simulate drawing 320 people 10 times and again record the 10 percents. Which set of 10 results is less variable? We expect the results of samples of size 320 to be more predictable (less variable) than the results of samples of size 20. That is “long-run regularity” showing itself.